M 325K
SPRING 2024

HW#10A, SEC 8.3
AND HW#10B; PART II, SEC 8.3

SECTION 8.3: In Problems #30, 31, 33 and 34, R is an equivalence relation on set A.

#30. To Prove: For all a in A, $a \in [a]$.

Proof: Let $a \in A$ be given.
Since R is reflexive, aRa.
$\therefore a \in [a]$ by definition of "$[a]$, the equivalence class of a".
$\therefore$ For all $a \in A$, $a \in [a]$, by Direct Proof.
QED.

#31

To Prove: For all a and b in A, if $b \in [a]$, then aRb.

Proof: Let $a, b \in A$ be given.
Suppose $b \in [a]$.
Then bRa, by def'n of "The class of a, $[a]$".
$\therefore aRb$, by the symmetric property of R.
QED, by Direct Proof.

ALSO, INCIDENTALLY, For all $a, b \in A$, if aRb, then $b \in [a]$.
Proof: Let $a, b \in A$ be given.
Suppose aRb. [NTS: $b \in [a]$]
$\therefore$ Since R is symmetric, bRa.
$\therefore b \in [a]$, by definition of $[a]$

So, For all $a, b \in A$, $b \in [a] \Leftrightarrow aRb$.

Section 8.3, #33

To Prove: For all a and b in A,
if $[a] = [b]$, then $a R b$.

Proof:

Let $a \in A$ and $b \in A$ be given.
Suppose $[a] = [b]$. [NTS: $a R b$]

Then $[a] \subseteq [b]$ by definition of set equality.
Since R is reflexive, $a R a$.
$\therefore$ $a \in [a]$, by defin of $[a]$.
$\therefore$ Since $[a] \subseteq [b]$, $a \in [b]$.
$\therefore$ $a R b$ by definition of $[b]$.

$\therefore$ For all a and b in A, if $[a] = [b]$, then $a R b$, by Direct Proof.

QED.

Section 8.3, #34

To Prove: For all a, b, and x in A,
if $a R b$ and $x \in [a]$, then $x \in [b]$.

Proof:

Let a, b and x be any elements in A.
Suppose $a R b$ and $x \in [a]$. [NTS: $x \in [b]$].
$\therefore$ $x R a$ by defn of $[a]$.
Since $x R a$ and $a R b$, $x R b$, since R is Transitive.
$\therefore$ $x \in [b]$ by definition of $[b]$.

QED, by Direct Proof.